GCE

Mathematics A

H230/02: Pure Mathematics and Mechanics

AS Level

Mark Scheme for June 2022

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

© OCR 2022

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

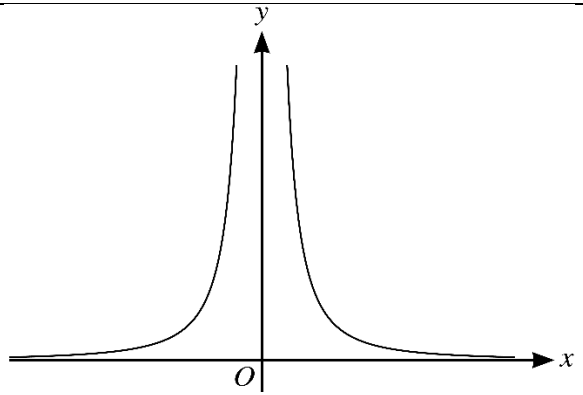
i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1			Critical values of x are $2, -3$	**B1**	**1.1**	Possibly seen in solution using set notation	
			$\{x : x < -3\} \cup \{x : x > 2\}$	**B1ft**	**2.5**	Follow through their two c.v. of x	$\pm 2, \pm 3$ only Curly brackets only With this notation only
				[2]			
2			**DR** $3x+1=4\sqrt{x}$				
			$3\left(\sqrt{x}\right)^2+1=4\sqrt{x}$ would be enough	**M1***	**1.1**	Recognise as a quadratic in $\sqrt{x}$	Condone not $=0$
			$3\left(\sqrt{x}\right)^2-4\sqrt{x}+1=0$	**A1**	**1.1**	All 3 terms on one side and $=0$	
			$\left(3\sqrt{x}-1\right)\left(\sqrt{x}-1\right)=0$	**M1dep***	**1.1**	Attempt to solve for $\sqrt{x}$	
			$x=1$ or $x=\frac{1}{9}$	**A1**	**1.1**		
			Alternative method $3x+1=4\sqrt{x}$				
			$(3x+1)^2=16x$, $9x^2+6x+1=16x$	**M1***	**1.1**	Square both sides	Three terms from squaring bracket, at least 2 correct and $=kx$ with $k=4$ or 16
			$9x^2-10x+1=0$	**A1**	**1.1**	All 3 terms on one side and $=0$	
			$(9x-1)(x-1)=0$	**M1dep***	**1.1**	Attempt to solve	
			$x=1$ or $x=\frac{1}{9}$	**A1**	**1.1**		
				[4]			

Question			Answer	Marks	AO	Guidance	
3			For example, let $x = \pi + 4, y = -\pi + 4$	M1	2.1	Choosing x and y such that **both** are irrational	May not see $x =, y =$
			$x + y = (\pi + 4) + (-\pi + 4) = 8$ which is rational (and hence the statement is disproved)	A1	2.2a	x and y chosen so that $x + y$ is rational. Must comment that answer is rational (oe) (and therefore the statement is disproved.)	
				[2]			

Question			Answer	Marks	AO	Guidance	
4			$x^2 + y^2 - 6x + 4y + k = 0$ $\Rightarrow (x-3)^2 - 9 + (y+2)^2 - 4 + k = 0$	M1*	1.1	Attempt to complete the square for both x and y terms. Must have $(x \pm 3)^2 + (y \pm 2)^2 + ...$	
			$(r^2 =) 9 + 4 - k = 5^2$	M1dep*	1.1	Setting up an equation for k correctly using either 5 or 5^2 (e.g., $\sqrt{13-k} = 5$ or $13 - k = 25$)	
			$k = -12$	A1 [3]	1.1	cao	

Question			Answer	Marks	AO	Guidance	
5	(a)	(i)	$y = \frac{k}{x}$ The curve passes through the point $\left(1, -\frac{1}{2}\right)$ so $k = -\frac{1}{2}$ or $y = -\frac{1}{2x}$ $y = -\frac{1}{2x} \Rightarrow y' = \frac{1}{2x^2}$	M1 A1 A1ft [3]	2.1 1.1 1.1	Allow any letter, (except x, y) or value, for k Allow this mark for just $-\frac{1}{2x}$ oe Differentiating their f(x) correctly Need to see their value of k substituted	 Need to see $y' =$ or $f'(x) =$ or $\frac{dy}{dx} =$
5	(a)	(ii)		B1ft [1]	1.1	Excellent curve in 1st and 2nd quadrants only: • Correct shape, symmetrical, not touching axis • Asymptote clearly the axes • Not finite Allow slight movement away from asymptote at one end but not more	Follow through provided their curve is of the form $y = \frac{k}{x^2}$ where $k > 0$
5	(b)		C has no stationary points as indicated by the fact that the curve for the gradient function (seen in part (a)(ii)) does not intersect (or touch) the x-axis	B1 [1]	2.4	Curve in **5(a)(ii)** must be of the form $y = \frac{k}{x^2}$	Need to see idea of intersecting, touching, crossing etc x-axis **only**

Question			Answer	Marks	AO	Guidance	
5	(c)		$y = -\frac{1}{2(x+2)}$	M1 A1 [2]	1.1 2.2a	Their $y = f(x)$ with x replaced by $x \pm 2$ oe (e.g., $y = -\frac{1}{2x+4}$)	May be $y = \frac{k}{x}$ Must have $y = \ldots$

Question			Answer	Marks	AO	Guidance	
6	(a)		$x = 1$	B1 [1]	1.1	cao or $(1, 0)$	Need not see $x =$
6	(b)		$\log_2\left(x-\frac{3}{2}\right)=-3$	M1	1.1	Setting $\log_2\left(x-\frac{3}{2}\right)+3$ equal to zero and isolating $\log_2\left(x-\frac{3}{2}\right)$ term	
			$x-\frac{3}{2}=2^{-3}$	M1	1.1	Correctly removing logs	After sensible work
			$x=2^{-3}+\frac{3}{2}\Rightarrow x=1.625$	A1 [3]	1.1	cao (o.e. exact answer e.g., $\frac{13}{8}$)	Need not see $x =$ Condone 1.63
6	(c)		$2\log_2 x=\log_2\left(x^2\right)$ seen	B1	1.1	Using the power law for logarithms	
			$\log_2\left(\frac{x^2}{x-\frac{3}{2}}\right)=3$	M1	1.1	Using subtraction or addition law for logarithms	oe, e.g. using $3 = \log_2 8$ etc
			$x^2=8\left(x-\frac{3}{2}\right)\Rightarrow x^2-8x+12(=0)$	M1	1.1	Removing logs correctly and re-arranging to a three-term quadratic in x	After sensible work
			$(x-2)(x-6)=0$ Therefore, the x coordinate of C is 2	A1 [4]	2.2a	**AG** so sufficient working must be shown	If solving **BC** then need to see $x = 2$ and $x = 6$ with $x = 2$ chosen as x coordinate

Question			Answer	Marks	AO	Guidance	
6	(d)		y-coordinate of C is 2	B1	1.1	soi	**NB** x-coordinate of C is 2
			Area $= \frac{1}{2}(1.625-1)(2)$	M1*	3.1a	Correct expression for the area of triangle ABC with their x-coordinate of B from **(b)** and their y-coordinate	0.625 Need $x = 1$
			$0.656 - 0.625$	M1dep*	1.1	Difference between their value and 0.656 is calculated	
			Under-estimate by 0.031 (units^2)	A1	3.2b	cao (both numerical value and 'under-estimate' required)	Allow 0.026 (comes from $x = 1.63$ in (b)) Allow 4.73% or 3.96%
				[4]			
7	(a)		$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (4\mathbf{i} - 3\mathbf{j}) - (2\mathbf{i} + 4\mathbf{j}) (= 2\mathbf{i} - 7\mathbf{j})$	M1	2.1	Correct method to find either $\overrightarrow{AB}$ or $\overrightarrow{BA}$	
			$\sqrt{53}$ or 53 from $\pm(2\mathbf{i} - 7\mathbf{j})$	A1	1.1	cao	
			$\lvert\overrightarrow{OA}\rvert = \sqrt{20}, \quad \lvert\overrightarrow{OB}\rvert = 5$	B1	1.1	Correct lengths for OA and OB (or their squares)	
			$\cos AOB = \dfrac{\left(\sqrt{20}\right)^2 + 5^2 - \left(\sqrt{53}\right)^2}{2\left(\sqrt{20}\right)(5)}$	M1	3.1a	Correct use of cosine rule for their OA, OB and AB	$\cos AOB$ may not be the subject, but substitutions must be correct for their values
			$\cos AOB = \left(\dfrac{20 + 25 - 53}{10\sqrt{20}} =\right) -\dfrac{4}{5\left(2\sqrt{5}\right)} = -\dfrac{2\sqrt{5}}{25}$	A1	2.2a	**AG** – sufficient working must be shown	Condone this result from calculator without intermediate working.

Question			Answer	Marks	AO	Guidance	
				[5]			
7	**(b)**		$\sin^2 AOB = 1 - \left(-\frac{2\sqrt{5}}{25}\right)^2$	**M1**	**3.1a**	Using the identity $\cos^2 X + \sin^2 X = 1$ with $\cos X = -\frac{2\sqrt{5}}{25}$	
			$\sin^2 AOB = \frac{121}{125} \Rightarrow \sin AOB = \frac{11\sqrt{5}}{25}$	**A1**	**1.1**	Or exact equivalent – justification not required for taking the positive square root	
				[2]			
7	**(c)**		Area of $OACB = 2\left(\frac{1}{2}\left(\sqrt{20}\right)(5)\left(\frac{11\sqrt{5}}{25}\right)\right)$	**M1**	**3.1a**	Use of $A = ab\sin C$ (or equivalent) with OA and OB and sin AOB	May not use exact values here
			22	**A1**	**1.1**	cao	Condone awrt 22.0
				[2]			

Question			Answer	Marks	AO	Guidance	
8	(a)		$f(x+h)=a(x+h)^2+b(x+h)$ $=a(x^2+2xh+h^2)+b(x+h)$	M1	2.1	Considers $f(x+h)$ and attempts to expand bracket squared	
			$f(x+h)-f(x)$ $=(ax^2+2ahx+ah^2+bx+bh)-(ax^2+bx)$ $=2xah+ah^2+bh$	A1	1.1	Correct simplified expression for $f(x+h)-f(x)$	
			$\frac{f(x+h)-f(x)}{h}=2ax+ah+b$	A1	1.1	Correct simplified expression	
			$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=2ax+b$	A1 [4]	2.2a	cao – must be explicit that the limit (and not simply $h = 0$) is considered	
8	(b)		$\int(ax^2+bx)\,dx=\frac{1}{3}ax^3+\frac{1}{2}bx^2(+c)$	M1*	2.1	Attempt to integrate (with at least one term correct)	
			$\int_1^4(ax^2+bx)\,dx=(\frac{64}{3}a+8b)-(\frac{1}{3}a+\frac{1}{2}b)(=21a+\frac{15}{2}b)$	M1dep*	1.1	Correct use of limits $x = 1$ and $x = 4$ in their integrated expression (need not be simplified)	
			$21a+\frac{15}{2}b=9$	M1	3.1a	Dependent on both previous M marks – setting up an equation in a and b using the area of shaded region	
			$(f'(4)=)8a+b=-0.75$	B1	1.1	Correct equation in a and b	
			$a=-0.375, b=2.25$	A1	1.1	**BC** (oe)	
			$y=-0.375x^2+2.25x$ with $x = 4$ gives $y = 3$ Equation of tangent: $y-3=-0.75(x-4)$	M1	1.1	Sets up the equation of the tangent at $x = 4$ using 4, –0.75 and their y value at $x = 4$ (dependent on all previous M marks) or for $\frac{\text{their } y}{k-4}=-0.75$	Equation of tangent may have y set to 0 and x equal to k
			$0-3=-0.75(k-4)\Rightarrow k=8$	A1 [7]	2.2a		

Question			Answer	Marks	AO	Guidance	
9			$\mathbf{F}+(3\mathbf{i}+2\mathbf{j})=4(-2\mathbf{i}+3\mathbf{j})$	M1	3.3	Using **F**=*m***a** with correct number of terms	Column vectors allowed
			$\mathbf{F}=4(-2\mathbf{i}+3\mathbf{j})-(3\mathbf{i}+2\mathbf{j})$ $\Rightarrow \mathbf{F}=-11\mathbf{i}+10\mathbf{j}$	A1 [2]	1.1	cao – isw if magnitude found	
10				M1	3.3	M1 for use of $s=ut+\frac{1}{2}at^2$ with $s=\pm 2, u=\pm 3.5$ and $a=\pm g$ and t is the total time for the motion	Possible to consider for eg motion up and motion down. In this case: M1 for **complete** method to find relevant times.
			$-2=3.5t+\frac{1}{2}(-9.8)t^2\ (\Rightarrow 4.9t^2-3.5t-2=0)$	A1	1.1	Correct equation – need not be simplified	A1 All equations correct – need not be simplified
			$t=1.09$	A1 [3]	1.1	**BC** (positive root only)	$t=1.0890679...$

Question			Answer	Marks	AO	Guidance	
11	(a)		$0\ (\text{m s}^{-2})$	B1 [1]	1.2		
11	(b)		**DR** $v = t\left(-t^2 + 11t - 24\right)$ $\Rightarrow v = -t^3 + 11t^2 - 24t$	B1	1.1	Expand and simplify v correctly	
			$\frac{\mathrm{d}v}{\mathrm{d}t} = -3t^2 + 22t - 24$	M1	3.4	Differentiate their cubic expression for v correctly	
			$3t^2 - 22t + 24 = 0$	M1	1.1	Sets their three-term quadratic in t equal to zero	
			$(3t - 4)(t - 6) = 0$	M1	1.1	Factorises (oe) their three-term quadratic in t	Condone this factorisation for $-3t^2 + 22t - 24 = 0$
			From sketch $T > 3$ therefore $T = 6$	A1 [5]	2.3	Correct value of T with reason for why $T \neq \frac{4}{3}$	Any working used to determine the required value of T must be accurate
11	(c)		$\int_0^3 v\mathrm{d}t$ and $\int_3^T v\mathrm{d}t$	M1	3.1b	Need to see attempt at integrals but may be **BC**	Where v is a cubic expression
			$(-)\frac{117}{4}$ and $\frac{261}{4}$	A1	1.1		
			Total distance $= \frac{117}{4} + \frac{261}{4} = 94.5\ (\text{m})$	A1 [3]	1.1	cao	

Question			Answer	Marks	AO	Guidance	
12	(a)		19.6 (N)	B1 [1]	3.4	cao oe ($2g$)	
12	(b)			M1*	3.3	Attempt at N2L for either P or Q - correct number of terms	M0 if mass includes g
			$8g - T = 8a$ $T - 6g = 6a$	A1	1.1	Correct equations for the motion of P and Q	
			$a = \frac{1}{8}(8g - T) \Rightarrow T - 6g = \frac{3}{4}(8g - T)$ $(\Rightarrow 4T - 24g = 24g - 3T)$	M1dep*	3.4	Eliminate a	Or if find a first then award this mark for an equation involving T only
			Magnitude of force exerted on pulley is $2T$	B1	3.1b	Either stated (anywhere in solution) or if 2(their T) seen	
			$T = \frac{48}{7}g \Rightarrow F = 134.4$ (N)	A1 [5]	1.1	Awrt 134(N)	
12	(c)		$a = 1.4$	B1	1.1	Correct a	$a = \frac{1}{7}g$
			$v^2 = 2(1.4)(1.75)\left(\Rightarrow v^2 = 4.9\right)$	M1*	3.4	Using $v^2 = u^2 + 2as$ with $u = 0$ to find speed of Q (or speed squared) after travelling 1.75	
			$0 = \left(\sqrt{4.9}\right)^2 + 2(-9.8)s \ \ (\Rightarrow s = 0.25)$	M1dep*	3.4	Using $v^2 = u^2 + 2as$ with $v = 0$ and $a = \pm g$	
			Total distance travelled by Q is 0.25 + 1.75 = 2 (m)	A1 [4]	3.2a	cao	Condone awrt 2.0
12	(d)		One factor could be the presence of air resistance	B1 [1]	3.5a	Any correct factor **B0** for 'use a more accurate value of g' If **more** than one factor given then **B1** if all correct. **B0** if not.	Friction String is not light String is not inextensible P and Q are not particles Wind speed

Need to get in touch?

If you ever have any questions about OCR qualifications or services (including administration, logistics and teaching) please feel free to get in touch with our customer support centre.

Call us on

01223 553998

Alternatively, you can email us on

support@ocr.org.uk

For more information visit

ocr.org.uk/qualifications/resource-finder

ocr.org.uk

Twitter/ocrexams

/ocrexams

/company/ocr

/ocrexams

CAMBRIDGE
UNIVERSITY PRESS & ASSESSMENT

OCR is part of Cambridge University Press & Assessment, a department of the University of Cambridge.

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored. © OCR 2022 Oxford Cambridge and RSA Examinations is a Company Limited by Guarantee. Registered in England. Registered office The Triangle Building, Shaftesbury Road, Cambridge, CB2 8EA.

Registered company number 3484466. OCR is an exempt charity.

OCR operates academic and vocational qualifications regulated by Ofqual, Qualifications Wales and CCEA as listed in their qualifications registers including A Levels, GCSEs, Cambridge Technicals and Cambridge Nationals.

OCR provides resources to help you deliver our qualifications. These resources do not represent any particular teaching method we expect you to use. We update our resources regularly and aim to make sure content is accurate but please check the OCR website so that you have the most up-to-date version. OCR cannot be held responsible for any errors or omissions in these resources.

Though we make every effort to check our resources, there may be contradictions between published support and the specification, so it is important that you always use information in the latest specification. We indicate any specification changes within the document itself, change the version number and provide a summary of the changes. If you do notice a discrepancy between the specification and a resource, please contact us.

Whether you already offer OCR qualifications, are new to OCR or are thinking about switching, you can request more information using our Expression of Interest form.

Please get in touch if you want to discuss the accessibility of resources we offer to support you in delivering our qualifications.